On the Paucity of Soluble Superelliptic Equations

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Abstract

Let $F \in \mathbb{Z}[x, z]$ be a homogeneous polynomial of degree $2n+1 \geq 5$. A result of Darmon and Granville known as "Faltings plus epsilon" states that when F is separable, the superelliptic equation $y^2 = F(x, z)$ has finitely many primitive integer solutions. In this talk, we state and sketch a proof of an asymptotic version of "Faltings plus epsilon" which states that in families of superelliptic equations of sufficiently large degree and having a fixed non-powerful leading coefficient, a positive proportion of members have no primitive integer solutions, and moreover that a positive proportion of members that have solutions over $\mathbb{Z}_p$ for every prime p do not have any solutions over $\mathbb{Z}$ and thus fail the Hasse principle. Our result can be viewed as an analogue for superelliptic equations of Bhargava's result that most even-degree hyperelliptic curves over $\mathbb{Q}$ have no rational points.